3CD Mathematics Specialist WACE Revision

**Mathematical Reasoning**

These challenging questions are taken from the NSW HSC Mathematics Extension Examinations and hence NOT ALL questions here may be suitable.

Machine generated alternative text: Question 8(15 marks) Use a SEPARATE writing booklet.
(a) (i) Using the substitution I tan-. or otherwise, show that
lOi O
cotO + -tan= -5.cot-.
(ii) Use mathematical induction to prove that. for integers n  I,
I X
—tan———cot—— 2cotx.
r=i 2’ 2F 2” 2
(iii) Show that
hm  ——an--=--2cotx.
r=I 2’ 2’ X
(iv) Hence find the exact value of
r I ir I ir
tan-i- + tan1 + (an.jg +

**Answers**

Machine generated alternative text: (AM. ¿ k- 4,14  =  J_ (st0j -.2 cx
—oc r.’
= MM 2...  — .2. —
— — Ltx.
v’- X. 
=
:— .2-_ ?_t9t.x
T
Pid- x-
s.
Uð’t :  4r - ? -
,-,oO ,-.-, -wMachine generated alternative text: (p)() c   L.
¿H t4 ct 2 c4x.= RI-IS
Ç (I) 4J(tA E 
u(r  2 t
t_j (Dt?E_
r:f
:ÌIL
j.
- J-  - It - .4- -J-- -Ñ 
¿i c,2’ ?‘-‘
-. 2eot
=,  —2ct*x.
bjû’) ---&
72 r€Q- AIiis b J-cJ Iwæu4
j

**Note there are no answers available for the following questions**

Machine generated alternative text: Question 8(15 marks) Use a SEPARATE writing booklet.
(a) It is given thai 2cosA smB = sin(A + B) — sin(A — B). (Do NOT prove this.)
Prove by induction thai for integers n  1,
sin 2n1
cos8+cos3O+...+cos(2n—I)t1=
2 sinO

Machine generated alternative text: Question 6 (15 marks) Use a SEPARATE writing booklet.
(a) (i) Use the binomial theorem
(a+ br = a + ()aT_1b + + b
(o show that. for n  2,
r
(ii) Hence show that, for n 2,
n+2 4n+8
< .
2’ n(n—1)
(iii Prove by induction that. for integers n  I.
, n—I
(Il ¡I ì ¡11 n+2
1+21—1+31—i +•••+ni—i =4—
‘2) ‘2) ‘2) 2’
(iv) Hence determine the limiting sum of the series
1+2(ij+3(4.)+....

Machine generated alternative text: Question 7 (continued)
(C) The sequence {  } is given by
4 + x
x=l and x = for nl.
1 i+I
(i) Prove by induction that foc n  1
I +
x =2
I-as
where a =
3
(ii) Hence find the limiting value of x as n —°°.

Machine generated alternative text: Question 6 (15 marks) Use a SEPARATE writing booklet.
(a) for each integer n  O, let J(x) = f tet di.
(i) Prove by induction that
F x x3
I(x) = n![l_e_x(1+x+÷+ ... +:_..J .
6i) Show that
O
Jo n+l
(iii) Hence show that
o  ... 1
‘... 1! 2! n! (n-+1)!
(iv) Hence find the limiting value of 1+!+J_+  as n —*°°.
1! 2! n!